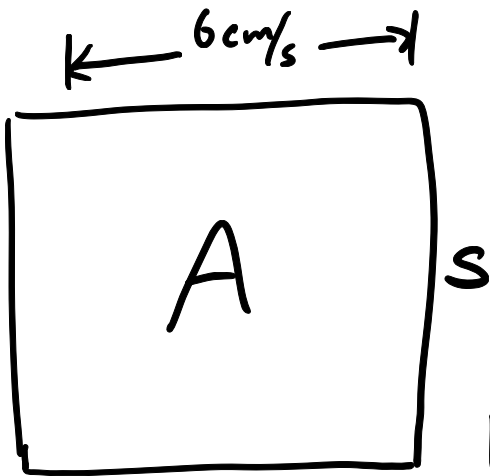


Related Rates

Ex: Each side of a square is increasing at a rate of 6 cm/s . At what rate is the area of the square increasing when the area is 16 cm^2 ?

Sol:



s = side length

A = area

Relation: $A = s^2$

Known quantities:

$$A = 16 \text{ cm}^2, \quad \frac{ds}{dt} = 6 \text{ cm/s}$$

Wanted quantity?

$$\frac{dA}{dt} = ?$$

$$\frac{d}{dt}(A = s^2)$$

$$\hookrightarrow \frac{dA}{dt} = 2s \frac{ds}{dt}$$

want ? known

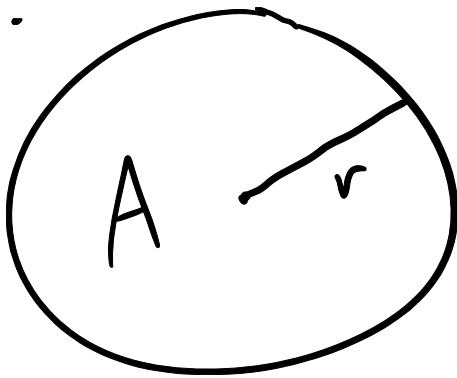
Solve for s: $A = 16 \text{ cm}^2$

$$16 \text{ cm}^2 = s^2 \Rightarrow s = 4 \text{ cm}$$

$$\frac{dA}{dt} = 2(4\text{cm})(6\text{cm/s}) = 48\text{cm}^2/\text{s}$$

Ex: The area of a circle is increasing at a rate of $3\text{cm}^2/\text{s}$. Find the rate of change of the radius of the circle when the circle has area $4\pi\text{cm}^2$.

Sol:



$r = \text{radius}$

$A = \text{area}$

Relation

$$A = \pi r^2$$

Known: $\frac{dA}{dt} = 3\text{cm}^2/\text{s}$

$$A = 4\pi = \pi r^2 \Rightarrow r = 2\text{cm}$$

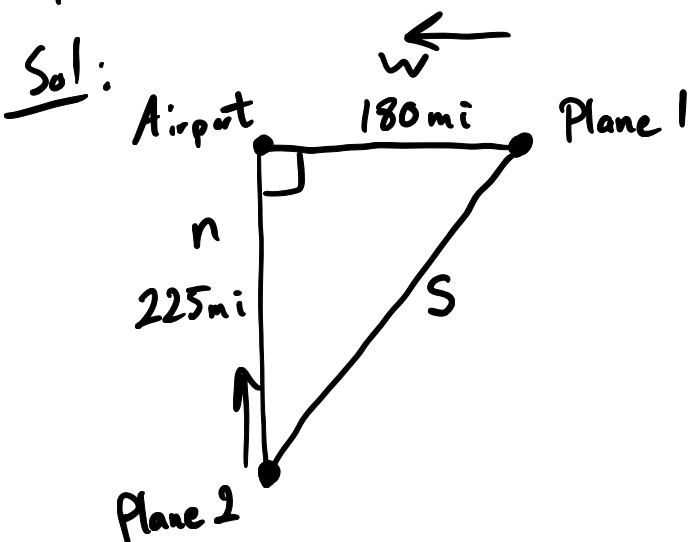
Want: $\frac{dr}{dt}$

$$\frac{d}{dt}(A = \pi r^2) \rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$3\text{cm}^2/\text{s} = 2\pi(2\text{cm}) \frac{dr}{dt}$$

$$\boxed{\frac{dr}{dt} = \frac{3}{4\pi} \text{cm/s}}$$

Ex: Two small planes approach an airport, one flying due west at 120 mi/hr and the other flying due north at 150 mi/hr . Assuming they fly at the same constant elevation, how fast is the distance between the planes changing when the westbound plane is 180 mi from the airport and the northbound plane is 225 mi from the airport?



Equation: $S^2 = n^2 + w^2$

Known: $w = 180 \text{ mi}$, $n = 225 \text{ mi}$

$$S = \sqrt{180^2 + 225^2} \text{ mi} \approx 288 \text{ mi}$$

$$\frac{dn}{dt} = -150 \text{ mi/hr}, \quad \frac{dw}{dt} = -120 \text{ mi/hr}$$

Want: $\frac{ds}{dt}$.

$$\frac{d}{dt}(S^2 = n^2 + w^2)$$

$$2s \frac{ds}{dt} = 2n \frac{dn}{dt} + 2w \frac{dw}{dt}$$

$$2(288 \text{ mi}) \frac{ds}{dt} = 2(225 \text{ mi})(-150 \text{ mi/hr}) + 2(180 \text{ mi})(-120 \text{ mi/hr})$$

$$\frac{ds}{dt} \approx -192 \text{ mi/hr}$$